Opener

Non-Calculator

$\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1-\ln x}{x^2}$ (E) $\frac{1+\ln x}{x^2}$

3-8 Derivatives of the Inverse Trig Functions

Learning Objectives:

I can calculate the derivatives of the inverse trig functions

I can calculate the derivatives of inverse functions given information about the function.

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Derivatives of Exponential Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Ex1. Find the Derivative

1.) $y = x^2 \sin^{-1} x$

1.) $y = x^2 \sin^{-1} x$

$\boxed{y' = 2x(\sin^{-1}x) + x^2\left(\frac{1}{\sqrt{1-x^2}}\right)}$

$f = x^2$ $\quad g = \sin^{-1}x$

$f' = 2x$ $\quad g' = \frac{1}{\sqrt{1-x^2}}$

2.) $y = \dfrac{\cos^{-1} x}{e^x}$

② $y = \dfrac{\cos^{-1}x}{e^x}$

$y' = \dfrac{\left[\frac{-e^x}{\sqrt{1-x^2}}\right] - e^x \cos^{-1}x}{(e^x)^2}$

$f = \cos^{-1}x$ $\quad g = e^x$

$f' = \frac{-1}{\sqrt{1-x^2}}$ $\quad g' = e^x$

3.) $y = \sin^{-1}(4x^2)$

$y = \sin^{-1} x$

$y' = \dfrac{1}{\sqrt{1-x^2}}$

$y' = \dfrac{1}{\sqrt{1-(4x^2)^2}} \cdot 8x$

$$\boxed{y' = \dfrac{8x}{\sqrt{1-16x^4}}}$$

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4.) $y = \cos^{-1}\left(\dfrac{2}{x}\right)$

$$y = \cos^{-1}\left(\frac{2}{x}\right) \qquad 2x^{-1}$$

$$y = \frac{-1}{\sqrt{1-\left(\frac{2}{x}\right)^2}} \cdot -2x^{-2}$$

$$= \frac{-1}{\sqrt{1-\frac{4}{x^2}}} \cdot \frac{-2}{x^2}$$

$$= \frac{2}{x^2\sqrt{\frac{x^2-4}{x^2}}} = \frac{2}{x^2\,\frac{\sqrt{x^2-4}}{x}} = \boxed{\frac{2}{x\sqrt{x^2-4}}}$$

5.) $y = \tan^{-1}(x^2e^x)$

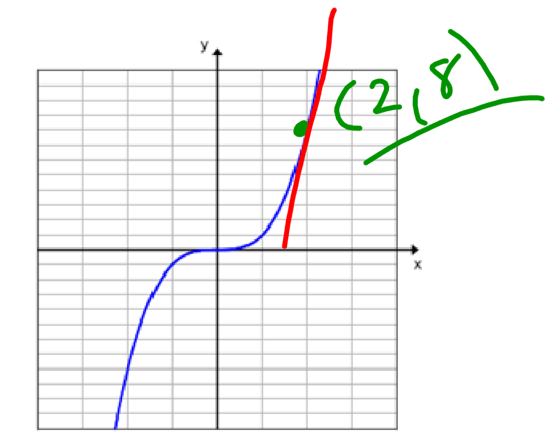
$$y' = \frac{1}{1+(x^2e^x)^2} \cdot \left[2xe^x + x^2e^x\right]$$

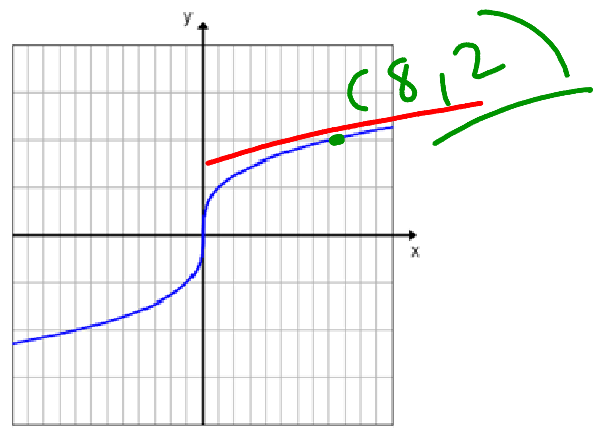
$$y' = \frac{2xe^x + x^2e^x}{1+x^4e^{2x}}$$

Inverse Functions

$y = f(x)$ $\qquad$ $y = f^{-1}(x)$

$y = x^3$ $\qquad$ $y = \sqrt[3]{x}$





$y' = 3x^2$

@ $x=2$ $\quad y' = 3(2)^2$

$y' = 12$

$y = x^{1/3}$

$y' = \frac{1}{3} x^{-2/3}$

$y' = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$

@ $x = 8$ $\quad y' = \frac{1}{3} \frac{1}{\sqrt[3]{8^2}}$

$y' = \frac{1}{3} \cdot \frac{1}{4}$

$y' = \frac{1}{12}$

Ex.1 Certain values of the function f(x) and its derivatives f'(x) are shown in the table below

x	f(x)	f'(x)
1	5	-3
2	1	-7
3	-8	-1/2

a.) Find $\frac{d}{dx}\left(f^{-1}(x)\right)$ @ x=1

$\underline{f}$ $\underline{f^{-1}}$

(, 1) (1 ,)

(2 , 1) (1 , 2)

slope = -7 slope = $-\frac{1}{7}$

b.) Find the equation of the tangent line to y = f^{-1}(x) @ x=1

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{7}(x - 1)$

Homework

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